

Towards Graphene-comprising Waveguide Resonators for **Kerr Comb Generation** in the **Non-Perturbative Electrodynamical Nonlinearity Regime**



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Introduction / Abstract

We present a study of **Kerr microcombs** [1] generated by CW pumping of **graphene-comprising silicon nitride waveguide ring resonators** in the NIR [2]. Our resonator is designed to access the **dissipative cavity soliton** regime under the combined effect of defocusing nonlinearity from graphene and normal group velocity dispersion (GVD) from a slot waveguide, properly accounting for the **wideband dispersion of all waveguide parameters**. We then proceed to study the effect of non-perturbative graphene nonlinearity on comb formation and efficiency.

Graphene-comprising Si-based Waveguides & Resonators

Silicon-based integrated waveguides provide high lateral confinement:

- ✓ **Miniaturization** of footprint
- ✓ Boosting of **nonlinear (NL)** effects

Graphene can be incorporated in such waveguides to enhance them with:

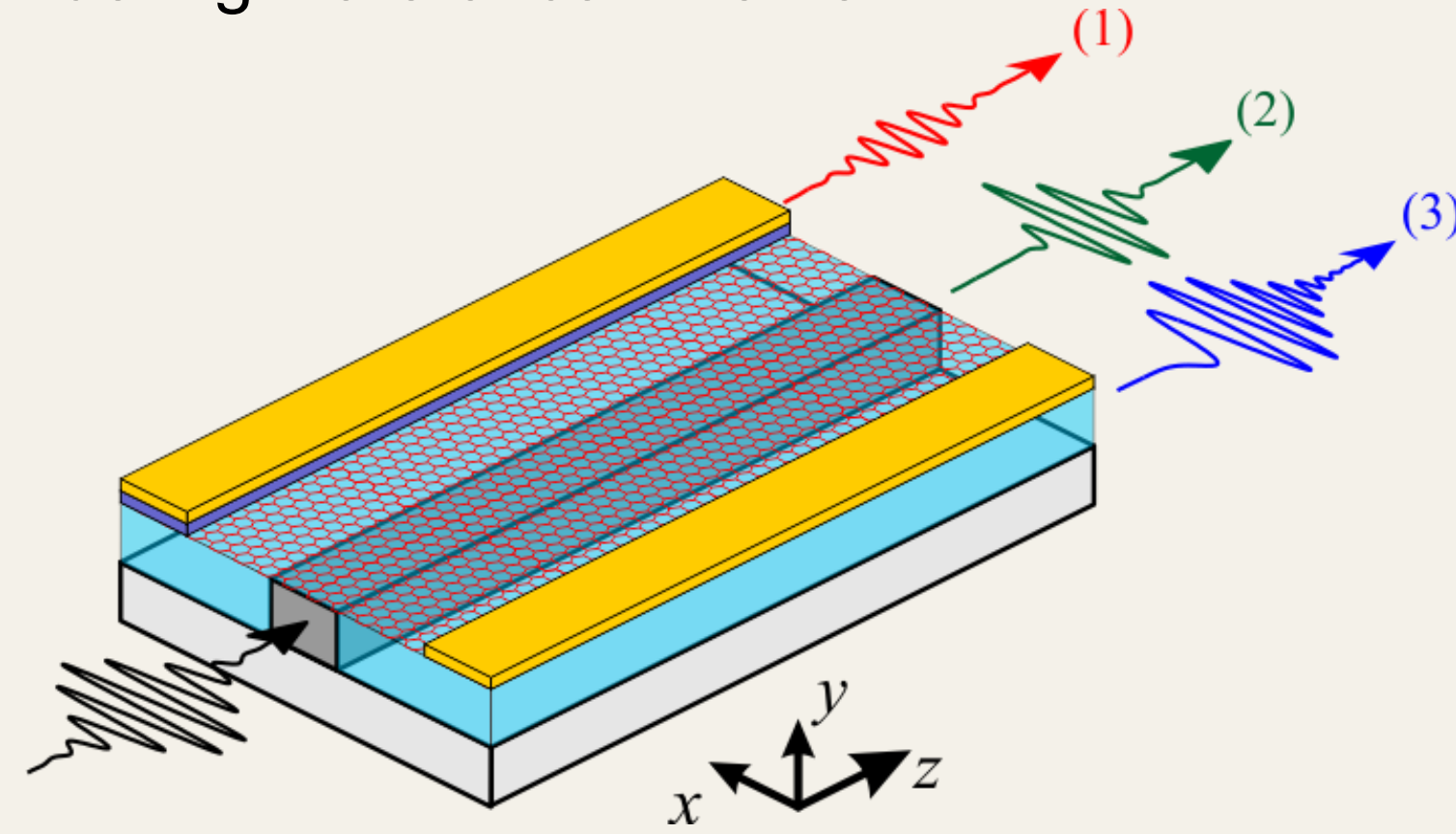
- ✓ Electro-optic tunability
- ✓ Rich and voltage-tunable NL
 - High 3rd-order (Kerr) response
 - Saturable absorption

Ultrafast NL functionalities

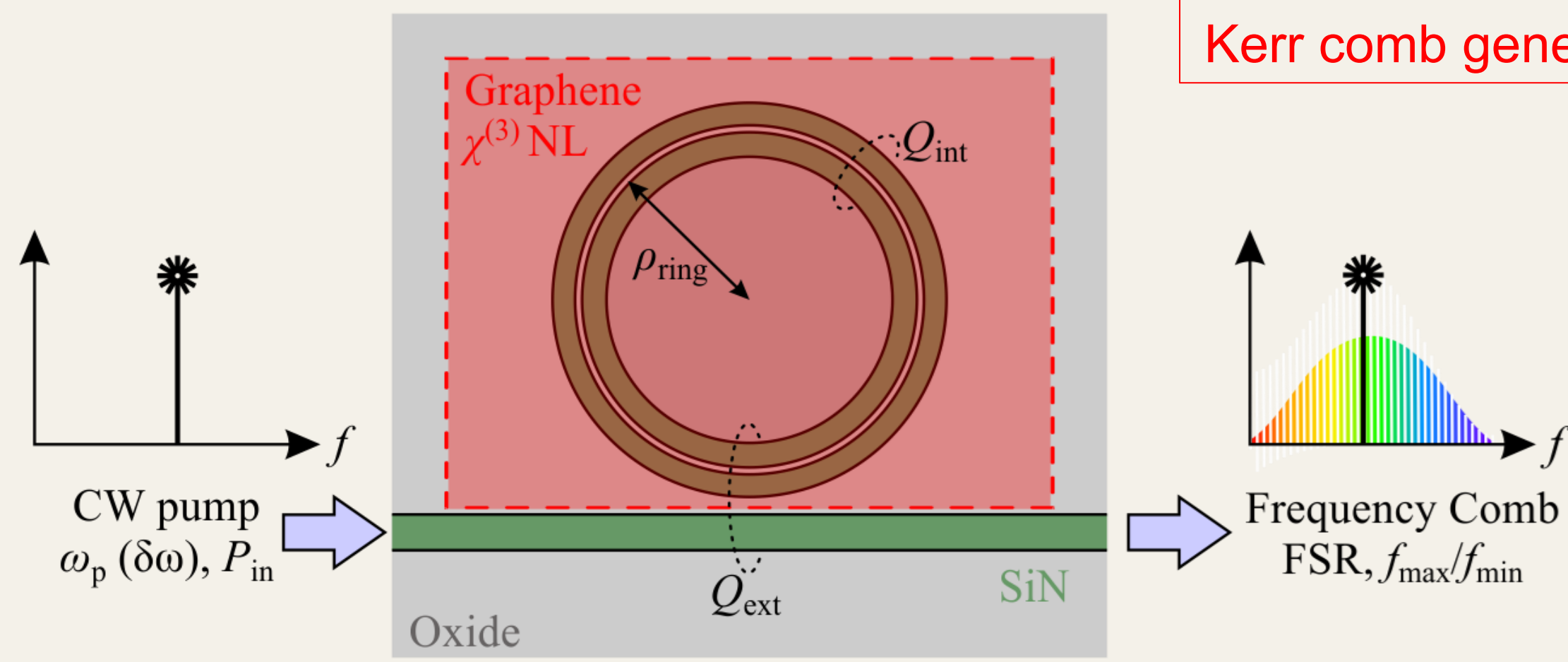
- Waveguide cross-section engineering
- Graphene quality and electrical tuning via its chemical potential (μ_c)
- Input pulse: power, duration, shape, chirp

Kerr micro-combs with graphene-comprising waveguide ring resonators (WRR)

- Traveling wave cavity with high 3rd-order nonlinearity
- Low losses (high Q_{int}) and critical coupling
- CW pumping with a (slow) frequency tuning



Graphene provides the nonlinearity for Kerr comb generation



Methods

Nonlinear Schrödinger Equation (NLSE)

- Pulse's slowly varying envelope $A(z, \tau)$ distortion along z -waveguide
- Waveguide effective parameters: $\{\alpha, \beta, \gamma_{\text{NL}}\} = \{\text{absorption, dispersion, Kerr-NL}\}$
- ❖ $\delta_{\text{GNL}}(z, \tau) = \text{Graphene NL} \rightarrow \text{Free-carrier effects (see GHEM)}$

$$\frac{\partial A}{\partial z} = \left[-\frac{\alpha}{2} + \left(\sum_{m=2}^{\infty} i^{m+1} \frac{\beta_m}{m!} \frac{\partial}{\partial \tau^m} \right) + i\gamma_{\text{NL}} |A|^2 - \delta_{\text{GNL}} \right] A = F_{\text{NLSE}}(z, \tau) A$$

Lugiato-Lefever Equation (LLE)

- Total field evolution inside a pumped dispersive nonlinear cavity
- $E = E(t_{\text{slow}}, \tau_{\text{fast}})$: two-timescale representation
- Similar to NLSE with $z \rightarrow t_{\text{slow}}$, but also driven ($\sqrt{\theta} E_{\text{in}}$) and detuned ($i\delta_0$)
- Other parameters: roundtrip time ($t_R = 1/\text{FSR}$), cavity length (L),

$$t_R \frac{\partial E}{\partial t} = L F_{\text{NLSE}}(t, \tau) E + (-0.5\theta - i\delta_0) E + \sqrt{\theta} E_{\text{in}}$$

Split-step Fourier Method (SSFM)

- Solve NLSE or LLE: Each "step" in z (NLSE) or t_{slow} (LLE) is split in two
- Spectral (Fourier) domain: Linear terms, e.g., absorption and dispersion
- Time domain: nonlinear terms, Kerr and/or free-carrier effects
- ❖ In-step iterations (typically 2-5) for convergence and stability

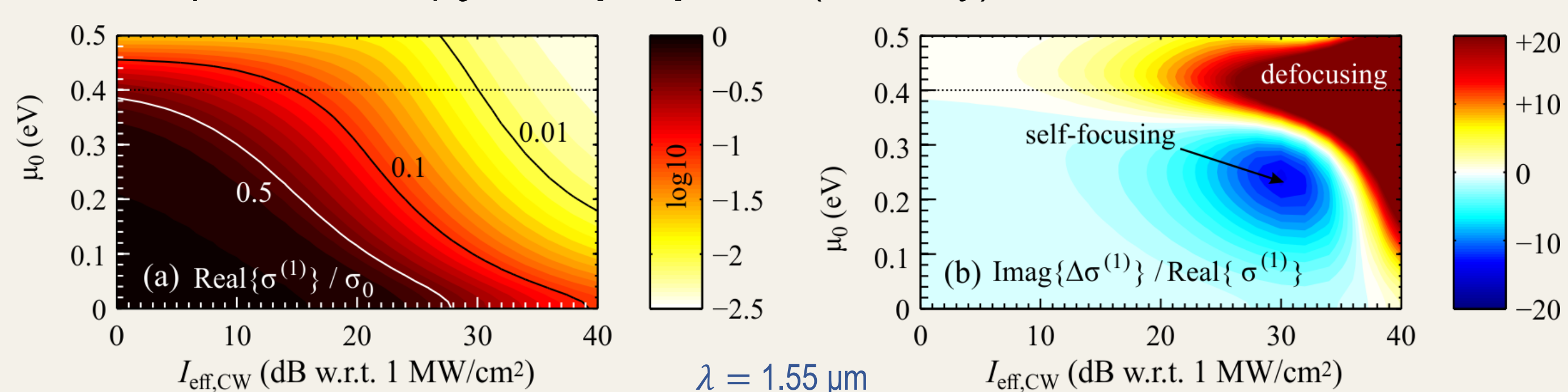
Material: Graphene Nonlinearity

Equilibrium: Carrier temperature = lattice temperature (T)

- Perturbative 3rd order NL response [3] \rightarrow Computation of $\sigma^{(3)} = \sigma^{(3)}(\lambda, \mu_c, T)$

Non-equilibrium: Graphene "hot electron" model (GHEM) [4]

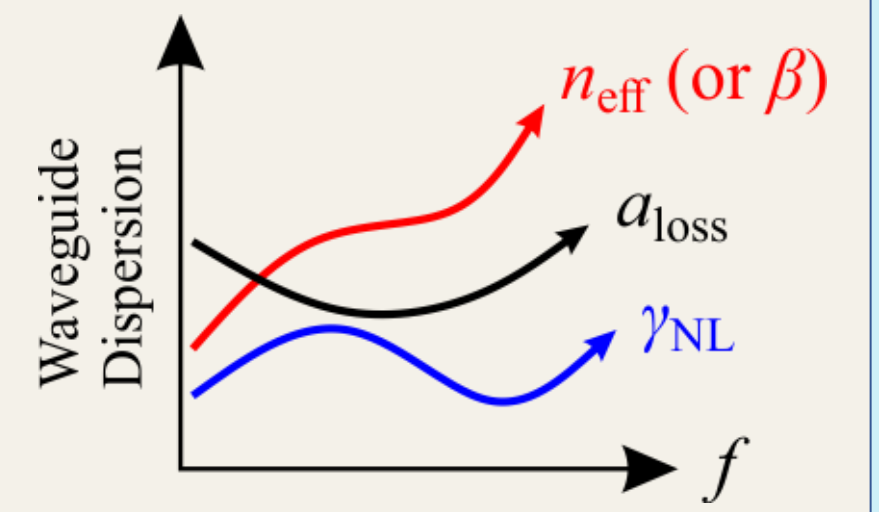
- Fermi-Dirac electrodynamic microscopic model \rightarrow Non-perturbative
- Complex photoconductivity $\Delta\sigma^{(1)} = \sigma_{\text{NL}}^{(1)} - \sigma_{\text{lin}}^{(1)}$: from photogenerated free-carrier (plasma) density and energy, and graphene conductivity (Kubo formulas)
 - Depends on (λ, μ_c, T) , **input power** (intensity), and GHEM lifetimes



Design: Waveguide for Graphene NL-based Kerr Combs

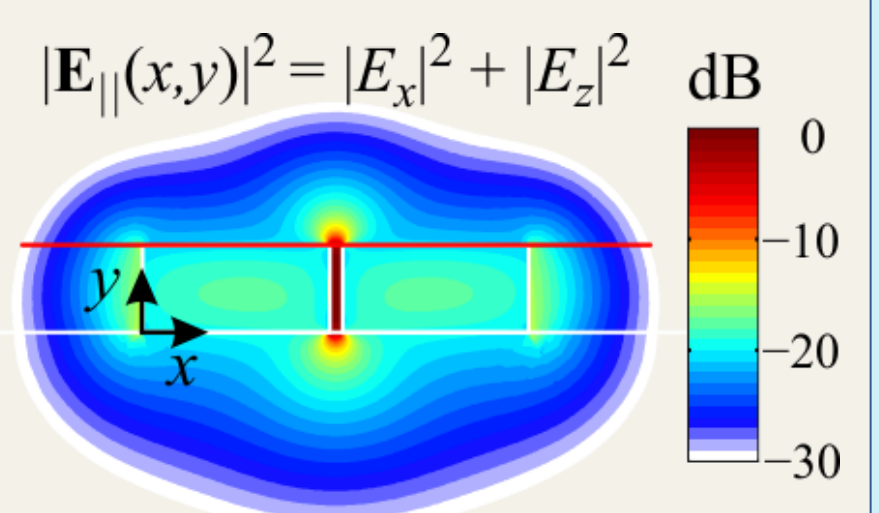
General Methodology

- Maximize overlap of E-field (tangential) with graphene
- Dispersion of $\{\alpha, \beta, \gamma_{\text{NL}}\}$ parameters \rightarrow FEM mode solver
- GHEM [4]: Define effective parameters to relate $\Delta\sigma^{(1)}$ to pulse power and NL refraction and absorption/bleaching.



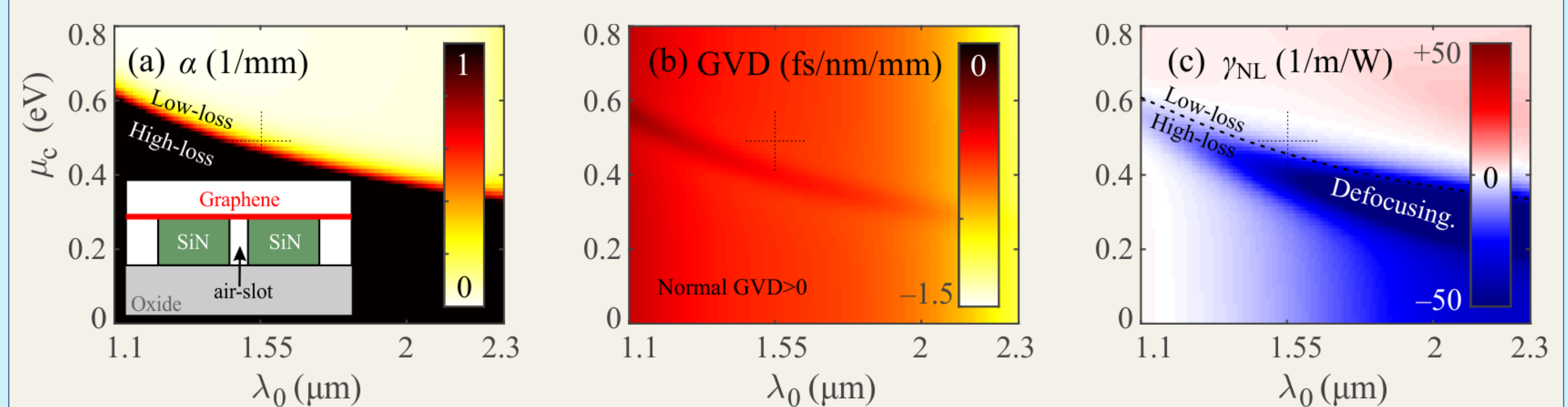
Graphene-based Kerr comb: The slot-waveguide

- Graphene nonlinearity is defocusing, $\gamma_{\text{NL}} < 0$
 - For cavity soliton (Kerr combs): normal GVD < 0
- Octave spanning around **1.55 μm** (1.1 to 2.3 μm)
 - Low-loss (transparency): Si-nitride on insulator
- Waveguide: 800 nm × 500 nm SiN rails; 50 nm air-slot



Graphene Tuning

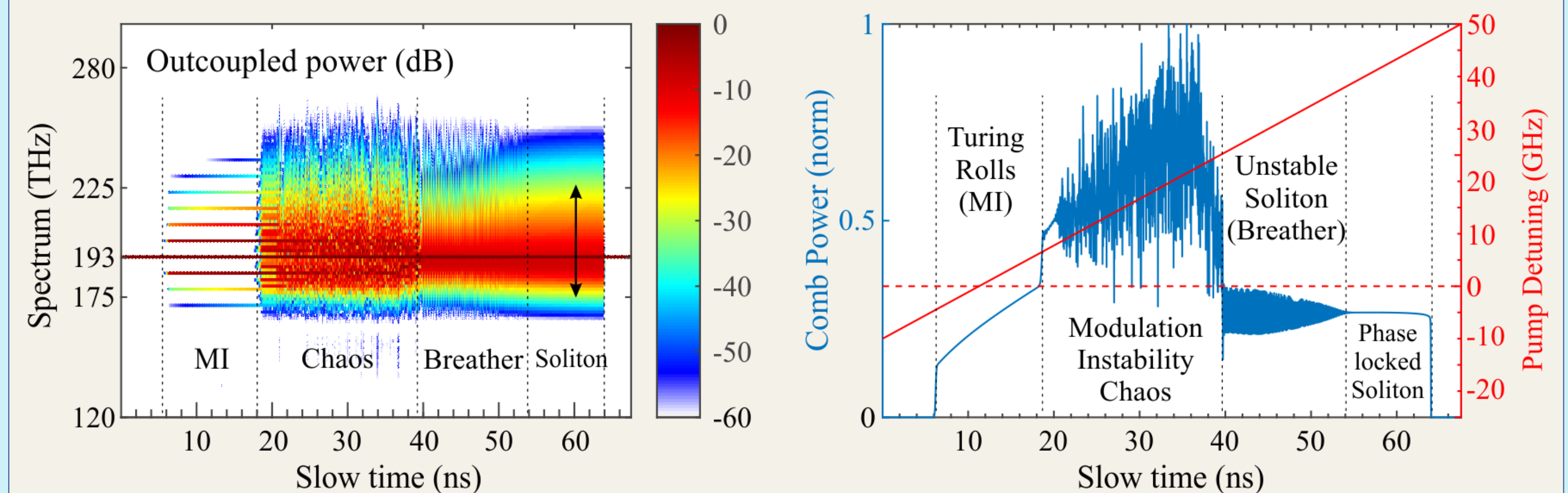
- Identify optimal graphene-monolayer chemical potential (μ_c)
- Frequency dispersion of w/g parameters $\{\alpha, \beta, \gamma_{\text{NL}}\}$ using $\sigma^{(3)}(\lambda, \mu_c, 300 \text{ K})$ [3]



Simulation Results: Kerr Comb Generation

Waveguide resonator and pumping laser

- CW laser pumping near 193 THz (1.55 μm) resonance
 - Power: $|E_{\text{in}}|^2 \approx 10 \text{ W}$
 - Detuning range (speed): $-10 \rightarrow +50 \text{ GHz}$ ($\sim 0.9 \text{ GHz/ns}$)
- Cavity length $L = 100 \mu\text{m} \rightarrow$ Roundtrip time: $t_R \sim 0.67 \text{ ps} \rightarrow \text{FSR} \sim 1.48 \text{ THz}$
- Graphene chemical potential $\mu_c \sim 0.5 \text{ eV}$ for 1.55 μm (see cross in figs above)
- $Q_{\text{ext}} \sim 3.6 \times 10^4$ (no dispersion) for critical coupling near 193 THz



Four regimes: (1) modulation instability [MI] "Turing rolls", (2) MI chaos, (3) unstable soliton "breathers", (4) stable solitons.

Conclusions & Outlook

- ✓ Kerr combs *can* be generated in graphene-comprising WRRs
 - ❖ Stability vs. pump power & detuning and vs. graphene's μ_c
- ✓ Dissipative cavity solitons originate from graphene's nonlinear $\text{Im}\{\sigma^{(3)}\}$
 - ❖ Full dispersion included, for the perturbative regime [3]
- ✓ **Voltage tuning** on graphene (μ_c) can control the comb through...
 - Absorption: $\text{Re}\{\sigma^{(1)}\} \rightarrow \alpha \rightarrow Q_{\text{int}}$
 - Refractive NL: $\text{Im}\{\sigma^{(3)}\} \rightarrow \gamma_{\text{NL}}$
 - ❖ In [2] graphene voltage tuning affected dispersion, not nonlinearity
- High cavity powers \rightarrow graphene non-perturbative NL regime (**GHEM**) [4]

References

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- [4] Pitilakis, A. and E. E. Kriezis, "Ultrafast pulse propagation in graphene-comprising nanophotonic waveguides considering nonperturbative electrodynamic nonlinearity," *JOSA B*, Vol. 39, No. 10, pp. 2723–2734, 2022.

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